ASSIGNMENT SET - I

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

B.Sc Hon.(CBCS)

Mathematics: Semester-III

Paper Code: C7T

[Numerical Analysis]

Answer all the questions

1. What is the degree of precision? Find the degree of precision of the Simpson's 1/3 rule.
2. For what values of α and β, the quadrature formula $\int_{-1}^{1} f(x)dx = \alpha f(-1) + f(\beta)$ is exact for all polynomials of degree ≤ 1.
3. Find the Weights w_1, w_2, w_3, so that the relation $\int_{-1}^{1} f(x)dx = w_1 f(-\sqrt{0.6}) + w_2 f(0) + w_3 f(\sqrt{0.6})$ is exact for the functions $f(x) = 1, x, x^2$.
4. Describe power method to find the largest (in magnitude) eigen-value and the corresponding eigen-vector of a matrix.

5. Write the convergence criteria at Newton-Raphson method to solve a non- linear equation.
6. Describe the iteration process to find the real root of function f(x) =0 in [a,b] and state the condition for which the iteration process converges with desire degree of accuracy . Find the condition of convergence of the iteration process. Show that order of convergence of iteration process is linear.
7. What do you mean by 'round –off errors' in numerical data? Show how these errors are propagated in a difference table?
8. Define k-th order forward difference of a function f(x). Prove that the n-th order forward difference of a polynomial of degree n is constant.
9. Describe the iteration process to find the solution of a system of non - homogeneous equation $Ax = B$ where A=$[a_{ij}]_{n \times n}$, X =$[x_1, x_2, \ldots, x_n]$ B = $[b_1, b_2, \ldots, b_n]$ and state its condition of convergence. What modification will be done if this condition is not satisfied?

10. If f(x) is a function such that f(0) + f(b)= -107 , f(1) + f(5) = -36 and f(2) +f(4)= -3, find f(3).

11. State general error formula for the functional relation u=f$(x_1, x_2, \ldots, x_n)$. Find the relative error of S where S=$\frac{a^2\sqrt{b}}{c^3}$ and a=6.54$\pm$0.01, b =48.64$\pm$0.02 $and\ c = 13.5 \pm 0.03$.

12. Write down the iterative formula of modified Euler method and state why it is better than Eular method.

13. Describe pivoting process to find the solution of non-homogeneous equation in Gauss's Elimination method.

14. Define 'degree of precision' of a numerical integration formula. What are it for Tapezoidal rule and Simpson's $\frac{1}{3}$ rule?

15. What is the principal for the numerical differentiation? Deduce the differentiation formulae for computing first interpolating point.

16. What are the basic differences between interpolation and appromimation?

17. Write down the sufficient conditions for the convergence of fixed point iteration method.

18. What are the advantages to approximate a function using orthogonal polynomials?

19. Describe approximation of a continuous function using orthogonal polynomials.

20. Let (2,2), (-1.5, -2), (4,4.5) and (-2.5,-3) be a sample. Use least squares method to fit the line y = a+bx based on this sample and estimate the total error.

21. Use least square method to solve the following system of equations: $x + 3y = 1$, $x - y = 5,\ -3x + y = 4,\ 3x + 2y = 7$.

22. What are the basic differences between interpolation and appromimation?

23. Discuss the Newton-Raphson method for a pair of non-linear equations with stated convergence conditions and convergences rate.

24. State the sufficient condition for convergence of the Gauss-Seidal iteration method to solve a system of non-linear equations containing three equations and three variables.

25. What are the advantages of Gaussian quadrature compared to Newton-Cote quadrature?

26. Use Newton-Rapshon method to solve the following system of non-linear equations: $f(x, y) = 0,\ g(x, y) = 0$.

27. Use Jacobi's method to determine all eigenvalues and the eigenvectors of the real symmetric matrix $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$.

28. Solve the system of equations

$$\begin{gathered} 2x + 4y - 2z = 14 \\ x + 3y - 4z = 16 \\ -x + 2y + 3z = 1 \end{gathered}$$

using LU-decomposition method.

29. Solve the boundary value problem $y'' + xy' + 4 = 0, y(0) = 0\ and y(1) = 0$ with step length $h = 0.25$.

30. Write the merits and demerits of the LU-decomposition method to solve a system of linear equations.

31. Suppose for a system of linear equations $AX = B$, the matrix A is decomposed as $A = LU$, where L and U are the lower and upper triangular matrices and they are known. Explain a suitable method to solve the equation $AX = B$ with the help of the matrices L and U.

32. Explain the Gauss-Lengendre quadrature formula for *n* nodes.

33. Describe Jacobi's method to find all eigenvalues and eigenvectors of a symmetric matrix.

34. Discuss fixed point iteration method to solve the following non-linear equations: f (x,y) = 0, g (x,y) = 0.

35. Find the largest eigenvalue in magnitude and the corresponding eigenvector of the matrix

$$\mathbf{A} = \begin{matrix} 2 & 3 & 1 \\ 4 & 3 & 4 \\ 7 & 6 & 1 \end{matrix}$$

36. Describe LU – decomposition method to solve a system of linear equations.

37. Using LU decomposition method solve the following system of equations:

$$2x + 3y - 6z = 2, \qquad 3x + y + 2z = 7, \qquad -3x + 2y + 6z = 7$$

38. Explain Gauss-Jacobi Iterative method to solve a system of linear equations.

39. Solve the following system of linear equations by Gauss-Jacobi Iterative method. $x + y + 4z = 9,\quad 8x - 3y + 2z = 20, 4x + 11y - z = 33$

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41. Analyze the stability of Euler's method for initial value ODE.

42. Distinguish predictor and corrector formulae in solving an ordinary differential equation.

43. Discussed Runge-Kutta 4th order method to solve a pair of first-orderordinary differential equations.

44. Solve the following system of equations $x = (8x - 4x^2 + y^2 + 1)/8$ and $y = (2x - x^2 + 4y - y^2 + 3)/4$ starting with $(x_0, y_0) = (1.1, 2.0)$, using Seidel iteration method.

45. Given $\frac{dy}{dx} = x - y^2$ with x=0, y=1. Find y(0.2) by second and fourth order Runge-Kutta methods.
46. Discuss the stability of second order Runge-Kutta method
47. The method $y_{n+1} = y_n + \frac{1}{4}(k_1 + 3k_2), n = 0,1,2,...$ where $k_1 = hf(x_n, y_n)$ and $k_2 = hf(x_n + \frac{2h}{3}, y_n + \frac{2k_1}{3})$ is used to solve the initial value problem $\frac{dy}{dx} = f(x, y) = -10y, y(0) = 1$. Then obtain the step size $\boldsymbol{h}$ for which the method will produce stable results.
48. Use fourth order Runge-Kutta method to solve the second order initial value problem $2y''(x) - 6y'(x) + 2y(x) = 4e^x$ with $y(0) = 1$ and $y'(0) = 1$ at $x = 0.2, 0.4$.
49. If a number be rounded up to n significant figures, then relative error is less than $\frac{1}{k \times K^{n-1}}$ where k is the first significant digit of the number.

$$i.e \quad E_R < \frac{1}{k \times K^{n-1}}$$

50. The maximum absolute error that occurs in rounding off a number after n places of decimal is $\frac{1}{2} \times 10^{-n}$.
51. The general formula of Newton Raphson method for finding $\boldsymbol{k^{th}}$real root R is $\boldsymbol{x_{n+1}} = \frac{(\boldsymbol{k-1})\boldsymbol{x_n^k} \quad \boldsymbol{+R}}{\boldsymbol{kx_n^{k-1}}}$.
52. Find a real root $f(x) = x^3 + 4x^2 - 10$ using the *Newton-Raphson* method for a root in [1, 2], correct up to five significant figures.
53. If Δ and ∇ are forward and backward difference operator respectively,

then find the value of $\Delta - \nabla$?

54. Show that the maximum error in linear interpolation is given by $\frac{(x_0 - x_1)^2 M}{8}$ where $M = \max | f'''(\xi)|, x_0 \le \xi \le x_1$

55. Solve by Modify Euler methods, the following differential equation Given $\frac{dy}{dx} = x^2 + y$ with x=0, y=1. Find y(0.01) and y(0.02) with step size h=0.0.

56. Using Taylor's series method of order 2 with the step size h=0.1, find the approximation value of y(0.1) for the differential equation $\frac{dy}{dx} = y^2 + x$ with x=0, y=1.

57. Evaluate $\int_0^1 \frac{dx}{1+x}$ by Simpson's 1/3 rule, taking h=0.1.

58. Describe the Newton's Fundamental interpolation formula by divided difference formula.

59. Using Newton's divided difference formula to find f(5) from the following table:

x:	0	2	3	4	7	8
f(x):	4	26	58	112	466	668

60. Find the cubic polynomial by Lagrange's interpolation formula that takes the following value;

x:	0	4	5	8
f(x):	1	2	1	10

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